3. ZÁPOČTOVÝ TEST
ČÁST A

4)

$$\frac{3i^{28}+i^{91}}{2+4i^{80}-6i^{21}} \Rightarrow \frac{3\cdot 1 + i^{88}\cdot i^{2}\cdot i^{1}}{2+4\cdot 1 - 6i^{20}\cdot i^{1}} = \frac{3-i}{6-6i} =$$

$$= \frac{3-i}{6-6i}\cdot\frac{6+6i}{6+6i} = \frac{18+18i-6i-6i^{2}}{36-36i^{2}} =$$

$$= \frac{18+12i+6}{72} = \frac{24+12i}{72} =$$

$$= \frac{1}{3}+\frac{1}{6}i$$

2)

$$x^2 - 4ix - 3 = 0$$

$$D = b^2 - 4ac$$

$$D = 16i^2 - 4 \cdot 1 \cdot (-3)$$

$$D = -16 + 12$$

$$D = -4$$

$$x_1, x_2 = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{4i + \sqrt{-4}}{2} = \frac{4i + 2i}{2} = \frac{2i + i}{} = 3i$$

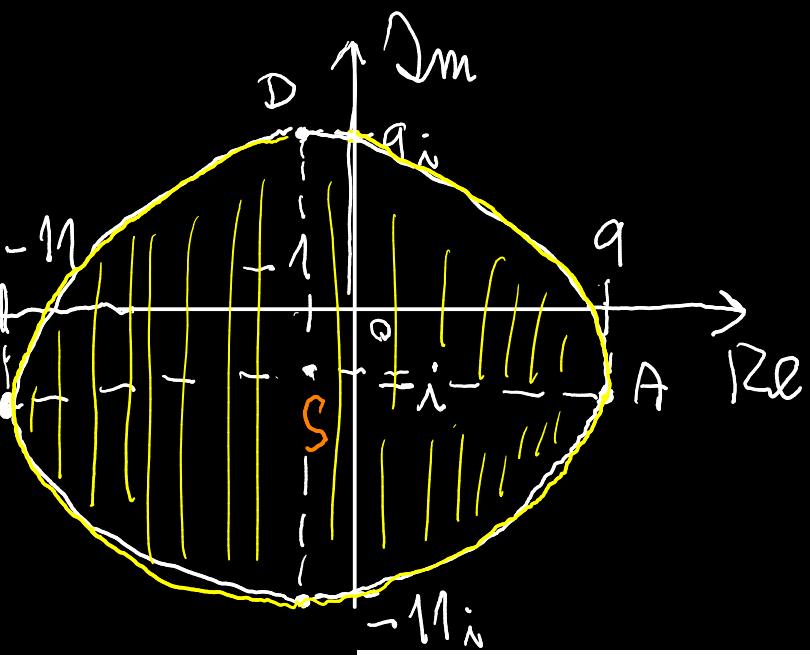
$$x_2 = \frac{4i - \sqrt{-4}}{2} = \frac{4i - 2i}{2} = \frac{2i}{} = i$$

3)

a) $|x+1+i| \leq |6+8i|$

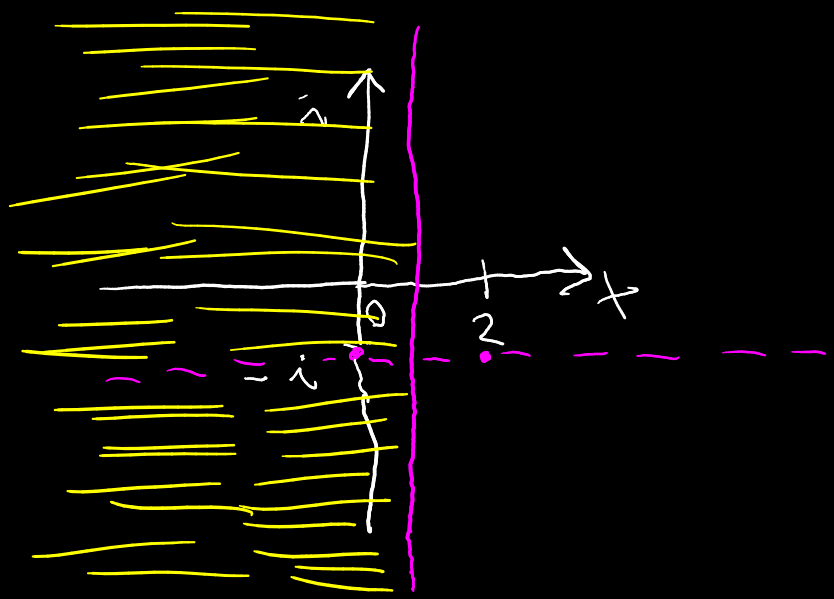
$\sqrt{36+64} = \sqrt{100} = 10$

A[9; -i] B[-11; -i]
C[-1; -11i] D[-1; 9i]



b) $|x-2+i| > |x+i|$

[2; -i] [0; -i]



4|

$$a = 2\sqrt{3} - 2i$$

$|a| = \sqrt{(2\sqrt{3})^2 + 2^2}$

$|a| = \sqrt{4 \cdot 3 + 4}$

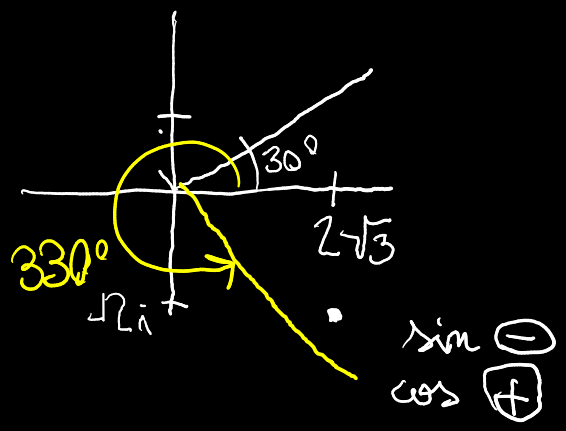
$|a| = 4$

$a = 4\left(\cos -\frac{\pi}{6} + i \cdot \sin -\frac{\pi}{6}\right)$

$a = 4e^{-\frac{\pi}{6}}$

$$\cos\varphi' = \frac{a}{|a|} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = 30° = \frac{\pi}{6}$$

$$\sin\varphi' = \frac{b}{|a|} = \frac{-2}{4} = -\frac{1}{2} = -30° = -\frac{\pi}{6}.$$



$$a = 2\sqrt{3} - 2i$$

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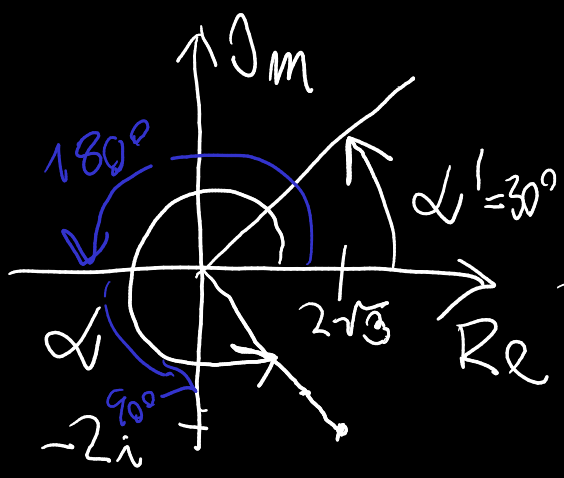
$|a| = \sqrt{4 \cdot 3 + 4}$

$|a| = 4$



$\alpha' = 30°$ $\cos\alpha = \frac{|a_1|}{|a|} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$= 30°$

$360 - 30 = 330°$

5

Umocněte b^{13} (výsledek uveďte v algebraickém tvaru) $b=3e^{i\frac{5\pi}{3}}$

$b=3e^{i\frac{5\pi}{3}} \Rightarrow b=3+\frac{5\pi}{3}i$

$|b|=\sqrt{3^2+\left(\frac{5\pi}{3}\right)^2}=\sqrt{9+\frac{25\pi^2}{9}}\doteq 6$

$\pi \ldots 180°$

$\frac{5\pi}{3} \ldots x°$

$x=\frac{180°\cdot\frac{5\pi}{3}}{\pi}$

$x=300°$

$b=6^{13}\left(\cos\frac{5\pi}{3}\cdot 13+i\sin\frac{5\pi}{3}\cdot 13\right)$

$b^{13}=6^{13}\left(\cos(300\cdot 13)+i\sin(300\cdot 13)\right)$

$b^{13}=6^{13}\left(\cos(3900°)+i\sin(3900°)\right)$

$b^{13}=6^{13}\left(\cos(300°)+i\sin(300°)\right)$

$b^{13}=6^{13}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

6)

$$(1+i).z = 2\bar{z} - i(1+i)$$

$(1+i)\cdot z = 2\bar{z} - i(1+i)$

$z = a+bi$

$\bar{z} = a-bi$

$z + iz = 2\bar{z} - i - i^2$

$a+bi+ai+bi^2 = 2a-2bi-i+1$

$a+ai+bi-b = 2a-2bi-i+1$

$-a-b+3bi+ai+i = -i+1$

$-a-b = 1$

$ai+3bi = -i$

$-a-b = 1$

$a+3b = -1$

$a = -1-3b$

$1+3b-b = 1$

$2b = 0$

$b = 0$

$b = 0$

$a = -1$

7

Vyřešte rovnici v R : $sin^2x - cos^2x + sinx = 0$

tg x = y
cotg x = 1/y

$$\sin^2 x - \cos^2 x + \sin x = 0$$
$$\sin^2 x - (1 - \sin^2 x) + \sin x = 0$$
$$2\sin^2 x + \sin x - 1 = 0 \Rightarrow t = \sin(x)$$
$$2t^2 + t - 1 = 0$$

$$D = b^2 - 4ac$$
$$D = 1 - 4 \cdot 2 \cdot (-1)$$
$$D = 1 + 8$$
$$D = 9$$

$$t_1 = \frac{-1+3}{4} = \frac{1}{2}$$
$$t_2 = \frac{-1-3}{4} = -1$$

$$\sin(x) = \frac{1}{2} = 30° = \frac{\pi}{6}$$
$$\sin(x) = -1 = 270° = \frac{3\pi}{2}$$
$$x = \frac{\pi}{6} + \frac{3\pi}{2} \cdot k ; k \in \mathbb{Z}$$

8

$R_1 = 1+3i\ \Omega \quad R_3 = 2+3i\ \Omega$

$R_2 = 2+i\ \Omega \quad R_4 = 3-4i\ \Omega$

$$Z = \frac{R_1}{R_2} + \frac{R_3}{R_4} = \frac{1+3i}{2+i} + \frac{2+3i}{3-4i} =$$

$$= \frac{1+3i}{2+i} \cdot \frac{2-i}{2-i} + \frac{2+3i}{3-4i} \cdot \frac{3+4i}{3+4i} =$$

$$= \frac{2-i+6i-3i^2}{4-i^2} + \frac{6+8i+9i+12i^2}{9-16i^2} =$$

$$= \frac{5+5i}{5} + \frac{-6+17i}{25} = \frac{25+25i-6+17i}{25} =$$

$$= \frac{19+42i}{25} = \left(\frac{19}{25} + \frac{42}{25}i\right)\Omega$$

$$\left|\frac{R_1}{R_2}\right| = \left|\frac{1+3i}{2+i}\right| = \frac{\sqrt{1^2+3^2}}{\sqrt{2^2+1^2}} = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$$

3. ZÁPOČTOVÝ TEST
ČÁST B

1

$$\frac{3+i^{31}}{-4+2i^{102}-8i^{16}} = \frac{3+i^{28}\cdot i^{3}}{-4+2i^{80}\cdot i^{20}\cdot i^{2}-8} = \frac{3-i}{-4-2-8} =$$

$$= \frac{3-i}{-14}$$

2

$$x^2 - 4xi - 20 = 0$$

$D = b^2 - 4ac$

$D = (-4i)^2 - 4\cdot(-20)$

$D = -16 + 80$

$D = 64$

$$x_1, x_2 = \frac{4i+8}{2} = 2i+4$$

$$\frac{4i-8}{2} = 2i-4$$

ZIC: x_1 ✓

$(2i+4)^2 - 4i(2i+4) - 20 = 0$

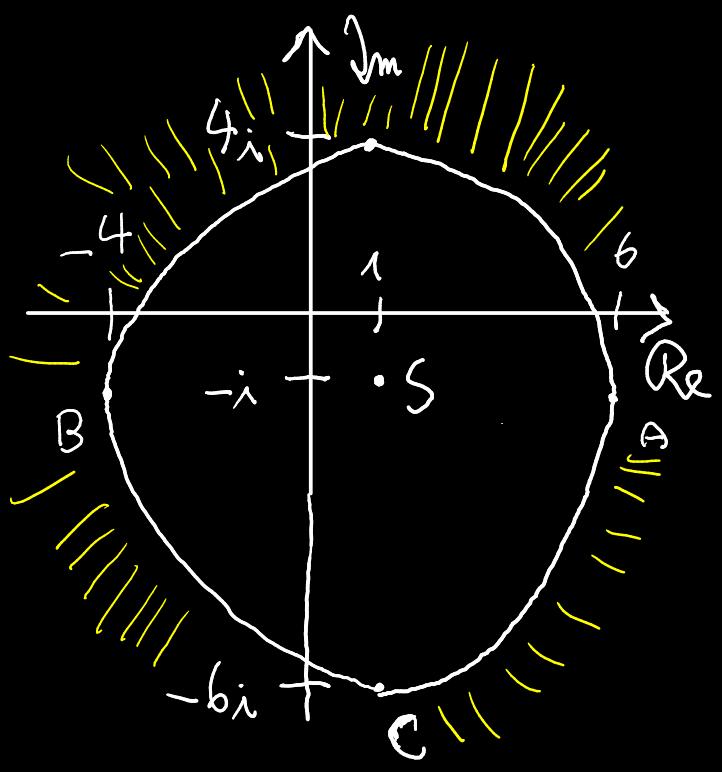
$4i^2 + 16i + 16 - 8i^2 - 16i - 20 = 0$

$-4 + 16i + 16 + 8 - 16i - 20 = 0$

$0 = 0$

$|x - 1 + i| > |3 - 4i|$

3| $S[1;-i] > 5 \quad \sqrt{a+16} = \sqrt{25} = 5$

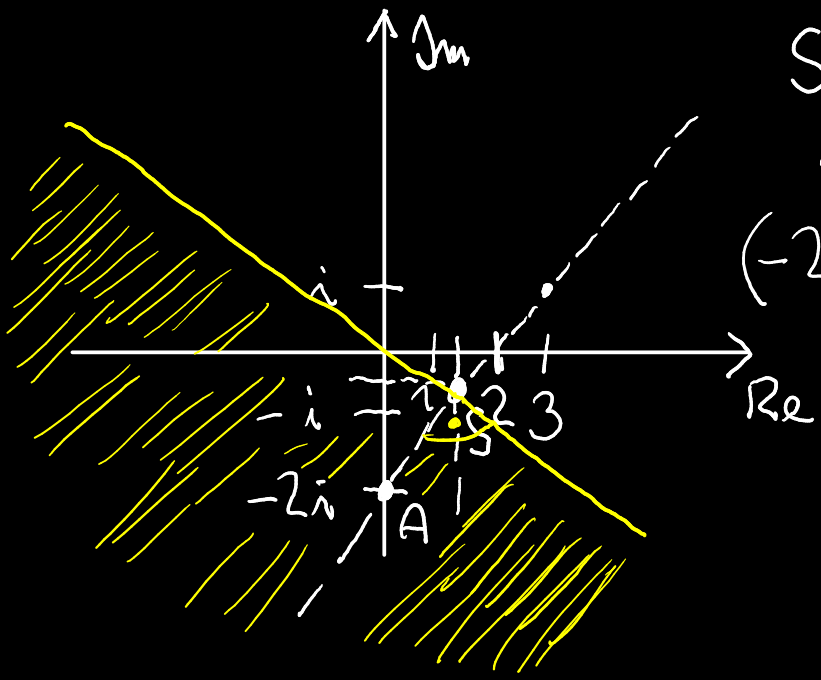


$A[6;-i]$ $C[1;-6i]$

$B[-4;-i]$ $D[1;4i]$

$|x+2i| \leq |x-3-i|$

$A[0;-2i] \leq B[3;i]$



$S[1{,}5;-0{,}5i]$

$3/2 = 1{,}5$

$(-2i+i)/2 = -\frac{1}{2} = -0{,}5i$

$(2+i).z = 2\bar{z} + i(2+i)$

41

$2z + iz = 2\bar{z} + 2i + i^2$

$2z + iz = 2\bar{z} + 2i - 1$

$z = a + bi$

$\bar{z} = a - bi$

$2(a+bi) + i(a+bi) = 2(a-bi) + 2i - 1$

$2a + 2bi + ai - b = 2a - 2bi + 2i - 1 \;/ -2a$

$ai + 4bi = 2i$

$-b = -1 \Rightarrow b = 1$

$a + 4b = 2$

$a = 2 - 4b$

$a = 2 - 4$

$a = -2$

$z = -2 + i$

$\bar{z} = -2 - i$

ZK:

$2z + iz = 2\bar{z} + 2i - 1$

$2(-2+i) + i(-2+i) = 2(-2-i) + 2i - 1$

$-4 + 2i - 2i - 1 = -4 - 2i + 2i - 1$

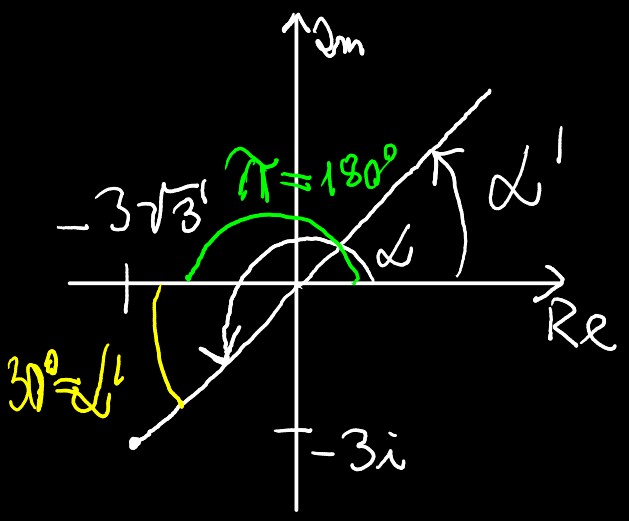
$-5 = -5$ ✓

5

$a = -3\sqrt{3} - 3i$

$|a| = \sqrt{(-3\sqrt{3})^2 + (3)^2}$

$= \sqrt{9 \cdot 3 + 9} = \sqrt{36} = 6$



$\cos\alpha' = \frac{|a_1|}{|a|} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} =$

$\alpha' = \frac{\pi}{6} = 30°$

$180° + 30° = 210° = \alpha$

6

$a = 6 \cdot \left(\cos\frac{7\pi}{6} + i \sin\frac{7\pi}{6}\right)$

$a = 6e^{\frac{7\pi}{6}i}$

$c = 3e^{i\frac{5\pi}{3}}$ $\quad c^{11} = ?$

$x = \frac{180 \cdot \frac{5\pi}{3}}{\pi}$

$x = 300°$

$\pi \ldots 180°$

$\frac{5\pi}{3} \ldots x$

$C = 3 \cdot \left(\cos\frac{5\pi}{3} + i \sin\frac{5\pi}{3}\right)$

$C^{11} = 3^{11} \cdot (\cos 300 \cdot 11 + i \sin 300 \cdot 11)$

$C^{11} = 3^{11} \cdot (\cos 3300° + i \sin 3300°)$

$c^{11} = 3^{11} \cdot (\cos 140° + i \sin 140°)$

$c^{11} = 3^{11} \cdot (-0{,}77 + 0{,}64i)$

$8 \cdot 360° = 2800°$

$3310 - 2800 = 500°$

$500° - 360° = 140°$

8

$2 \cdot \sin\left(4x + \frac{\pi}{3}\right) = -\sqrt{3}$

$\sin\left(4x + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$4x + \frac{\pi}{3} = \frac{4\pi}{3} + 2k\pi, \; k \in \mathbb{Z}$

$4x + \frac{\pi}{3} = \frac{5\pi}{3} + 2k\pi, \; k \in \mathbb{Z}$

$x = \frac{\pi}{4} + \frac{k\pi}{2}; \; k \in \mathbb{Z}$

$x = \frac{\pi}{3} + \frac{k\pi}{2}; \; k \in \mathbb{Z}$

RANDOM PŘÍKLADY K 3. TESTU

1)

$$\frac{3i^{17} + i^{80}}{2 + 4i^{29} - 8i^{21}} = \frac{3i^{16} \cdot i^{1} + i^{80} \cdot i^{10}}{2 + 4 - 8i^{29} \cdot i^{1}} = \frac{3i - 1}{2 + 4 - 8i} = \frac{3i - 1}{6 - 8i}$$

$$\frac{3i-1}{6-8i}\cdot\frac{6+8i}{6+8i}=\frac{18i+24i^2-6-8i}{36+64}=\frac{10i-30}{100}=-\frac{3}{10}+\frac{1}{10}i$$

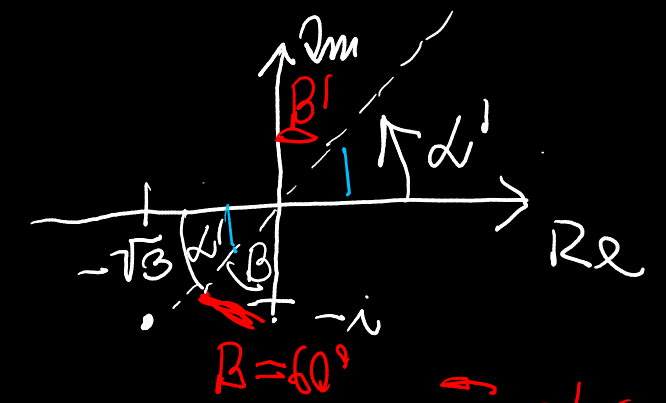
$$\sqrt{a^2+b^2}$$

$$a=-\sqrt{3}-i$$

$$|a|=\sqrt{(-\sqrt{3})^2+(-1)^2}$$

$$|a|=\sqrt{3+1}$$

$$|a|=\sqrt{4}=2$$



β' 90° − 30° = 60°

$$\cos\alpha'=\frac{|a_1|}{|a|}=\frac{\sqrt{3}}{2}=30°$$

$$\alpha=180°+30°=210°$$

$$-\frac{5\pi}{6}+2\pi=\frac{12\pi-5\pi}{6}=\frac{7\pi}{6}=\frac{6}{6}\pi+\frac{1}{6}\pi$$