

3. ZÁPOČTOVÝ TEST
ČÁST A

11

$$\frac{3i^{28} + i^{91}}{2 + 4i^{80} - 6i^{21}}$$

$$\begin{aligned} &\Rightarrow \frac{3 \cdot 1 + i^{88} \cdot i^2 \cdot i^1}{2 + 4 \cdot 1 - 6i^{20} \cdot i^1} = \frac{3 - i}{6 - 6i} \\ &= \frac{3 - i}{6 - 6i} \cdot \frac{6 + 6i}{6 + 6i} = \frac{18 + 18i - 6i - 6i^2}{36 - 36i^2} \\ &= \frac{18 + 12i + 6}{72} = \frac{24 + 12i}{72} \\ &= \frac{1}{3} + \frac{1}{6}i \end{aligned}$$

2

$$\underline{x^2 - 4ix - 3 = 0}$$

$$D = b^2 - 4ac$$

$$D = 16i^2 - 4 \cdot 1 \cdot (-3)$$

$$D = -16 + 12$$

$$D = -4$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$\nearrow x_1 = \frac{4i + \sqrt{-4}}{2} = \frac{4i + 2i}{2} = \frac{6i}{2} = 3i$

$\searrow x_2 = \frac{4i - \sqrt{-4}}{2} = \frac{4i - 2i}{2} = \frac{2i}{2} = i$

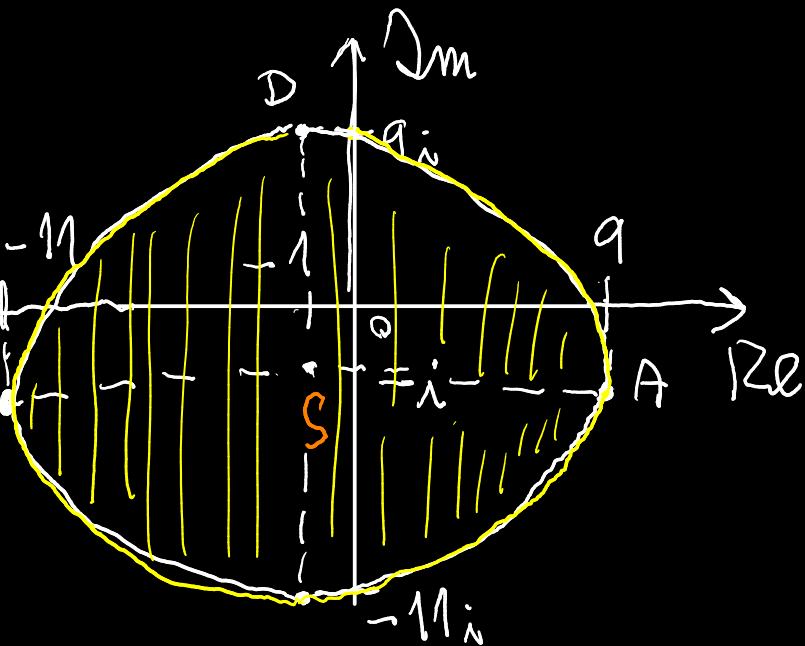
31

$$a) |x + 1 + i| \leq |6 + 8i|$$

$$\sqrt{36 + 64} = \sqrt{100} = 10$$

$$A [9; -i] \quad B [-11; -i]$$

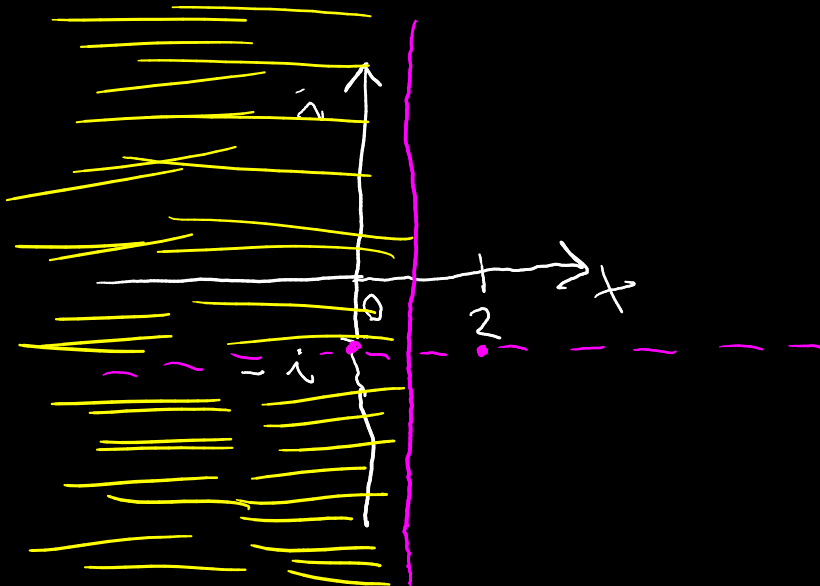
$$C [-1; -11i] \quad D [-1; 9i]$$



$$b) |x - 2 + i| > |x + i|$$

$$[2; -i]$$

$$[0; -i]$$



4

$$a = 2\sqrt{3} - 2i$$

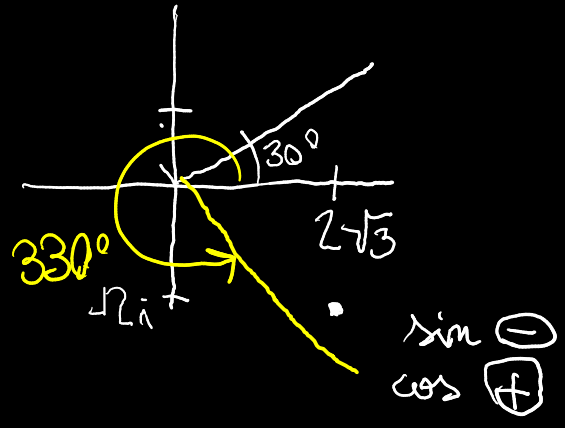
$$|a| = \sqrt{(2\sqrt{3})^2 + 2^2}$$
$$|a| = \sqrt{4 \cdot 3 + 4}$$
$$|a| = 4$$

$$a = 4 \left(\cos -\frac{\pi}{6} + i \sin -\frac{\pi}{6} \right)$$

$$a = 4 e^{-\frac{\pi}{6}}$$

$$\cos \varphi' = \frac{a}{|a|} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = 30^\circ = \frac{\pi}{6}$$

$$\sin \varphi' = \frac{b}{|a|} = \frac{-2}{4} = -\frac{1}{2} = -30^\circ = -\frac{\pi}{6}$$

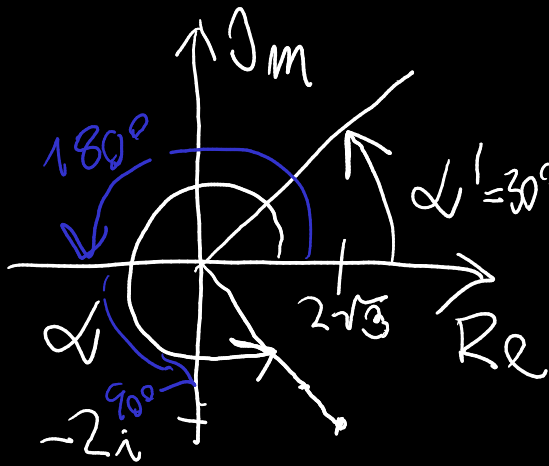


$$a = 2\sqrt{3} - 2i$$

$$|a| = \sqrt{(2\sqrt{3})^2 + 2^2}$$

$$|a| = \sqrt{4 \cdot 3 + 4}$$

$$|a| = 4$$



$$\cos \alpha = \frac{|a_1|}{|a|} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$
$$\alpha = 30^\circ$$

$$360 - 30 = 330^\circ$$

Umocněte b^{13} (výsledek uveďte v algebraickém tvaru) $b=3e^{i\frac{5\pi}{3}}$

$$b = 3e^{i\frac{5\pi}{3}} \Rightarrow b = 3 + \frac{5\pi}{3}i$$

$$|b| = \sqrt{3^2 + \left(\frac{5\pi}{3}\right)^2} = \sqrt{9 + \frac{25\pi^2}{9}} \approx 6$$

$$\begin{aligned} \pi &\dots 180^\circ \\ \frac{5\pi}{3} &\dots x^\circ \\ x &= \frac{180^\circ \cdot \frac{5\pi}{3}}{\pi} \\ x &= 300^\circ \end{aligned}$$

$$b = 6^{13} \left(\cos \frac{5\pi}{3} \cdot 13 + i \sin \frac{5\pi}{3} \cdot 13 \right)$$

$$b^{13} = 6^{13} \left(\cos(300 \cdot 13) + i \sin(300 \cdot 13) \right)$$

$$b^{13} = 6^{13} \left(\cos(3900^\circ) + i \sin(3900^\circ) \right)$$

$$b^{13} = 6^{13} \left(\cos(300^\circ) + i \sin(300^\circ) \right)$$

$$b^{13} = 6^{13} \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

6

$$(1+i).z = 2\bar{z} - i(1+i)$$

$$(1+i).z = 2\bar{z} - i(1+i)$$

$$z = a + bi$$

$$\bar{z} = a - bi$$

$$z + iz = 2\bar{z} - i - i^2$$

$$a + bi + ai + bi^2 = 2a - 2bi - i + 1$$

$$a + ai + bi - b = 2a - 2bi - i + 1$$

$$\underline{-a - b + 3bi + ai + i = -i + 1}$$

$$\underline{-a - b = 1}$$

$$\underline{ai + 3bi = -i}$$

$$\underline{-a - b = 1}$$

$$\underline{a + 3b = -1}$$

$$a = -1 - 3b$$

$$b = 0$$

$$a = -1$$

$$1 + 3b - b = 1$$

$$2b = 0$$

$$b = 0$$

Vyřešte rovnici v \mathbb{R} : $\sin^2 x - \cos^2 x + \sin x = 0$

$\operatorname{tg} x = y$
 $\operatorname{cotg} x = 1/y$

$$\sin^2 x - \cos^2 x + \sin x = 0$$

$$\sin^2 x - (1 - \sin^2 x) + \sin x = 0$$

$$2 \sin^2 x + \sin x - 1 = 0 \Rightarrow \Delta = \sin(x)$$

$$2 \Delta^2 + \Delta - 1 = 0$$

$$D = b^2 - 4ac$$

$$D = 1 - 4 \cdot 2 \cdot (-1)$$

$$D = 1 + 8$$

$$D = 9$$

$$\Delta_1 = \frac{-1 + 3}{4} = \underline{\underline{\frac{1}{2}}}$$

$$\Delta_2 = \frac{-1 - 3}{4} = \underline{\underline{-1}}$$

$$\sin(x) = \frac{1}{2} = 30^\circ = \frac{\pi}{6}$$

$$\sin(x) = -1 = 270^\circ = \frac{3\pi}{2}$$

$$x = \frac{\pi}{6} + \frac{3\pi}{2} \cdot k; k \in \mathbb{Z}$$

8

$$R_1 = 1 + 3i \, \Omega \quad R_3 = 2 + 3i \, \Omega$$

$$R_2 = 2 + i \, \Omega \quad R_4 = 3 - 4i \, \Omega$$

$$Z = \frac{R_1}{R_2} + \frac{R_3}{R_4} = \frac{1+3i}{2+i} + \frac{2+3i}{3-4i} =$$

$$= \frac{1+3i}{2+i} \cdot \frac{2-i}{2-i} + \frac{2+3i}{3-4i} \cdot \frac{3+4i}{3+4i} =$$

$$= \frac{2-i+6i-3i^2}{4-i^2} + \frac{6+8i+9i+12i^2}{9-16i^2} =$$

$$= \frac{5+5i}{5} + \frac{-6+17i}{25} = \frac{25+25i-6+17i}{25} =$$

$$= \frac{19+42i}{25} = \left(\frac{19}{25} + \frac{42}{25}i \right) \Omega$$

$$\left| \frac{R_1}{R_2} \right| = \left| \frac{1+3i}{2+i} \right| = \frac{\sqrt{1^2+3^2}}{\sqrt{2^2+1^2}} = \frac{\sqrt{10}}{\sqrt{5}} = \underline{\underline{\sqrt{2}}}$$

3. ZÁPOČTOVÝ TĚST

ČÁST B

1

$$\frac{3+i^{31}}{-4+2i^{102}-8i^{16}}$$

$$= \frac{3+i^{28} \cdot i^3}{-4+2i^{80} \cdot i^{20} \cdot i^2 - 8} = \frac{3-i}{-4-2-8} = \frac{3-i}{-14}$$

2

$$x^2 - 4xi - 20 = 0$$

$$D = b^2 - 4ac$$

$$D = (-4i)^2 - 4 \cdot (-20)$$

$$D = -16 + 80$$

$$D = 64$$

$$x_{1,2} = \frac{4i + 8}{2} = 2i + 4$$

$$\frac{4i - 8}{2} = 2i - 4$$

$z \in \mathbb{C}: \textcircled{x_1} \checkmark$

$$(2i+4)^2 - 4i(2i+4) - 20 = 0$$

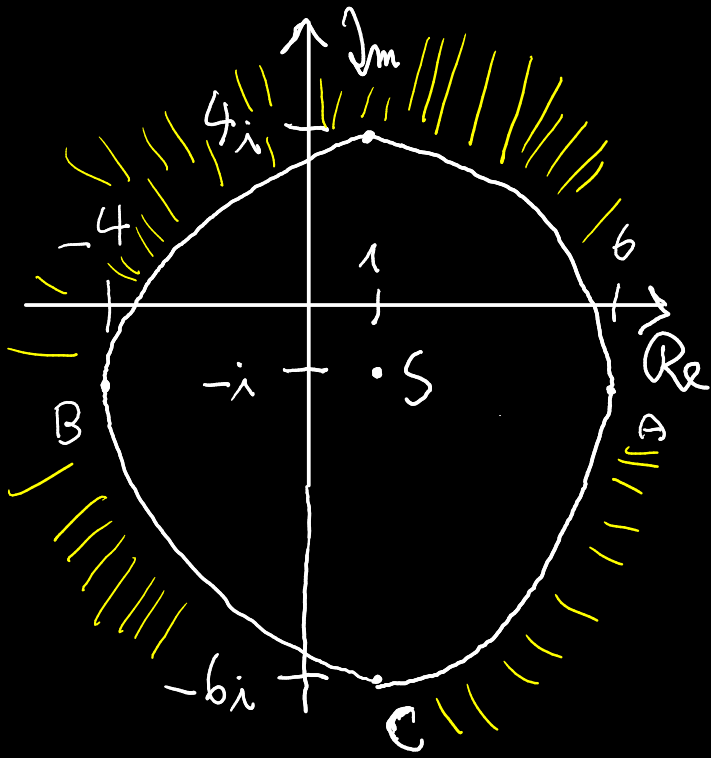
$$4i^2 + 16i + 16 - 8i^2 - 16i - 20 = 0$$

$$\underbrace{-4} + \underbrace{16i} + \underbrace{16} + \underbrace{8} - \underbrace{16i} - \underbrace{20} = 0$$

$$\underline{\underline{0=0}}$$

$$|x - 1 + i| > |3 - 4i|$$

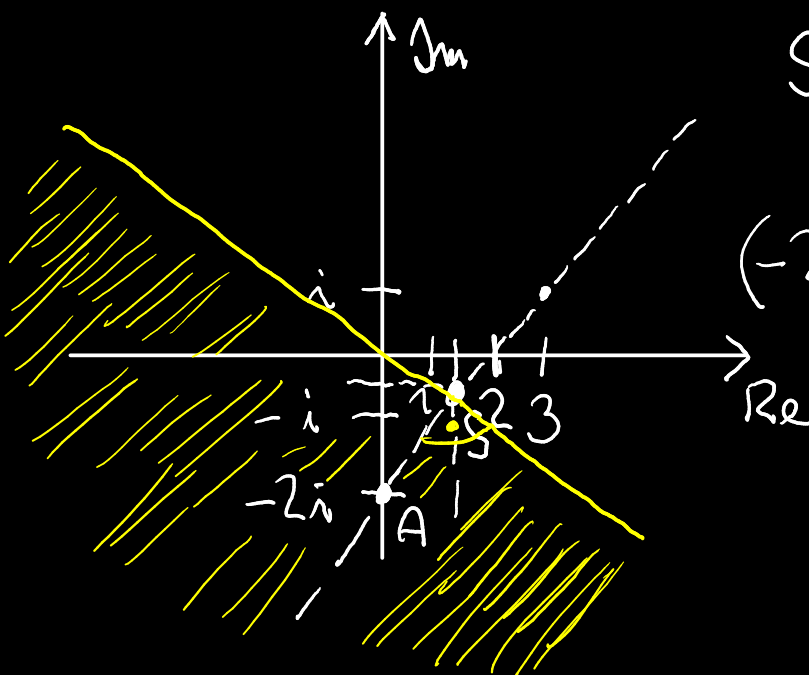
3 | $S[1; -i] > 5 \quad \sqrt{a+16} = \sqrt{25} = 5$



A[6; -i] C[1; -6i]
 B[-4; -i] D[1; 4i]

$|x + 2i| \leq |x - 3 - i|$

A[0; -2i] ≤ B[3; i]



S[1,5; -0,5i]
 $3/2 = 1,5$
 $(-2i + i)/2 = -\frac{1}{2} = -0,5i$

$(2 + i).z = 2\bar{z} + i(2 + i)$

41

$$2z + iz = 2\bar{z} + 2i + i^2$$

$$2z + iz = 2\bar{z} + 2i - 1$$

$z = a + bi$
$\bar{z} = a - bi$

$$2(a+bi) + i(a+bi) = 2(a-bi) + 2i - 1$$

$$\cancel{2a} + 2bi + ai - b = \cancel{2a} - 2bi + 2i - 1 \quad | -2a$$

$$ai + 4bi = 2i$$

$$-b = -1 \Rightarrow \underline{\underline{b = 1}}$$

$$a + 4b = 2$$

$$a = 2 - 4b$$

$$a = 2 - 4$$

$$\underline{\underline{a = -2}}$$

$$\underline{\underline{z = -2 + i}}$$

$$\underline{\underline{\bar{z} = -2 - i}}$$

ZK:

$$2z + iz = 2\bar{z} + 2i - 1$$

$$2(-2+i) + i(-2+i) = 2(-2-i) + 2i - 1$$

$$-4 + 2i - 2i - 1 = -4 - 2i + 2i - 1$$

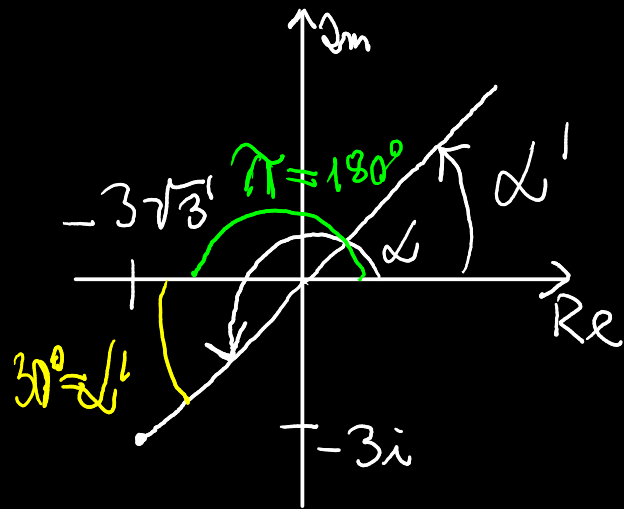
$$\underline{\underline{-5 = -5 \checkmark}}$$

5

$$a = -3\sqrt{3} - 3i$$

$$|a| = \sqrt{(-3\sqrt{3})^2 + (-3)^2}$$

$$= \sqrt{9 \cdot 3 + 9} = \sqrt{36} = 6$$



$$\cos \alpha' = \frac{|a_1|}{|a|} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\alpha' = \frac{\pi}{6} = 30^\circ$$

$$180^\circ + 30^\circ = 210^\circ = \alpha$$

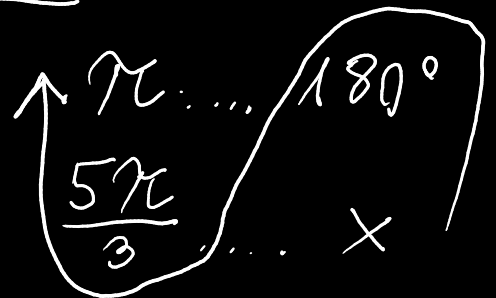
6

$$a = 6 \cdot \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$$

$$a = 6 e^{\frac{7\pi}{6} i}$$

$$x = \frac{180 \cdot \frac{5\pi}{3}}{\pi}$$

$$x = 300^\circ$$



$$c = 3e^{i\frac{5\pi}{3}} \quad C^{11} = ?$$

$$C = 3 \cdot \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)$$

$$C^{11} = 3^{11} \cdot (\cos 300 \cdot 11 + i \sin 300 \cdot 11)$$

$$C^{11} = 3^{11} \cdot (\cos 3300^\circ + i \sin 3300^\circ)$$

$$C^{11} = 3^{11} \cdot (\cos 140^\circ + i \sin 140^\circ)$$

$$8 \cdot 360^\circ = 2880^\circ$$

$$C^{11} = 3^{11} \cdot (-0,77 + 0,64i)$$

$$\begin{array}{r} 3300 \\ - 2880 \\ \hline \end{array}$$

$$500^\circ$$

$$- 360^\circ$$

$$\hline 140^\circ$$

8

$$2 \cdot \sin\left(4x + \frac{\pi}{3}\right) = -\sqrt{3}$$

$$\sin\left(4x + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$4x + \frac{\pi}{3} = \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

$$4x + \frac{\pi}{3} = \frac{5\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

$$x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

$$x = \frac{\pi}{3} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

RANDOM PŘÍKLADY K 3. TESTU

$$\begin{aligned} 1) \frac{3i^{17} + i^{90}}{2 + 4i^{20} - 8i^{21}} &= \frac{3i^{16} \cdot i^1 + i^{80} \cdot i^{10}}{2 + 4 - 8i^{20} \cdot i^1} = \frac{3i^{-1} + i^{-1}}{2 + 4 - 8i} = \frac{3i^{-1} + i^{-1}}{6 - 8i} \end{aligned}$$

$$\frac{3i-1}{6-8i} \cdot \frac{6+8i}{6+8i} = \frac{18i+24i^2-6-8i}{36+64} = \frac{10i-30}{100} = \underline{\underline{-\frac{3}{10} + \frac{1}{10}i}}$$

$$a = -\sqrt{3} - i$$

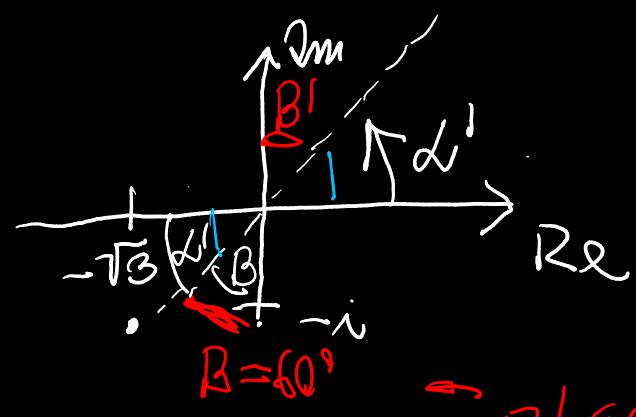
$$|a| = \sqrt{(-\sqrt{3})^2 + (-1)^2}$$

$$|a| = \sqrt{3+1}$$

$$|a| = \sqrt{4} = 2$$

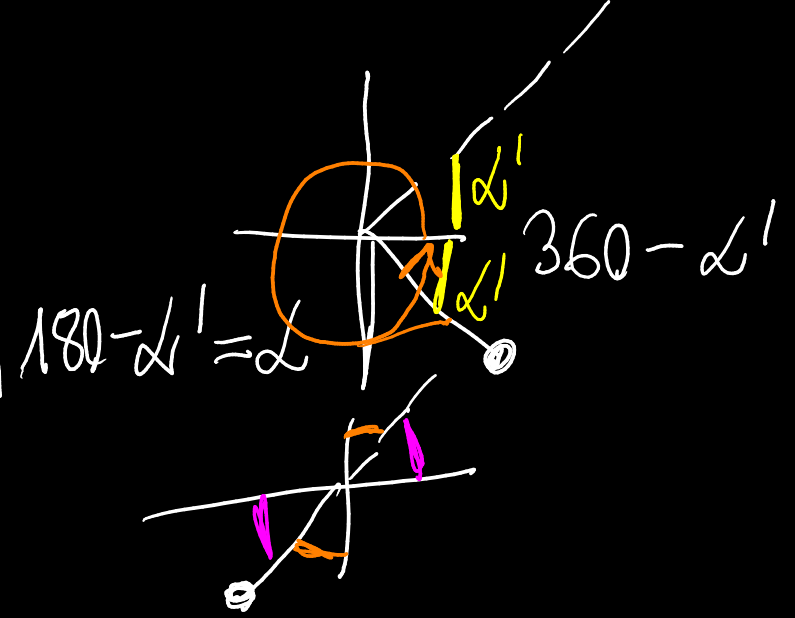
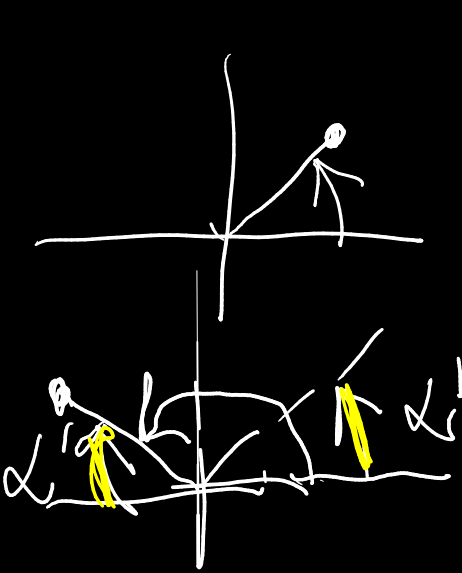
$$\cos \alpha' = \frac{|a_r|}{|a|} = \frac{\sqrt{3}}{2} = 30^\circ$$

$$\alpha = 180^\circ + 30^\circ = 210^\circ$$



$$\rightarrow B' \quad 90^\circ - 30^\circ = 60^\circ$$

$$-\frac{5\pi}{6} + 2\pi = \frac{12\pi - 5\pi}{6} = \frac{7\pi}{6} = \frac{6}{6}\pi + \frac{1}{6}\pi$$



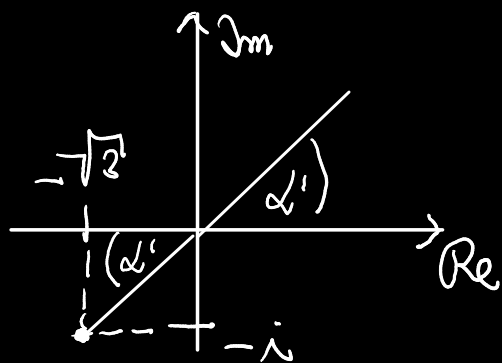
$$a = -\sqrt{3} - i$$

$$|a| = \sqrt{a^2 + b^2}$$

$$|a| = \sqrt{3 + 1} = 2$$

$$\cos \alpha' = \frac{|a_1|}{|a|} = \frac{\sqrt{3}}{2} = 30^\circ$$

$$\alpha = 180^\circ + 30^\circ = 210^\circ$$



$$a = 2 \cdot (\cos 210^\circ + i \sin 210^\circ) \quad \left| \begin{array}{l} \pi = 180^\circ \\ \pi = 30^\circ \end{array} \right.$$

$$a = 2 e^{\frac{7\pi}{6} i}$$

$$\frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$$

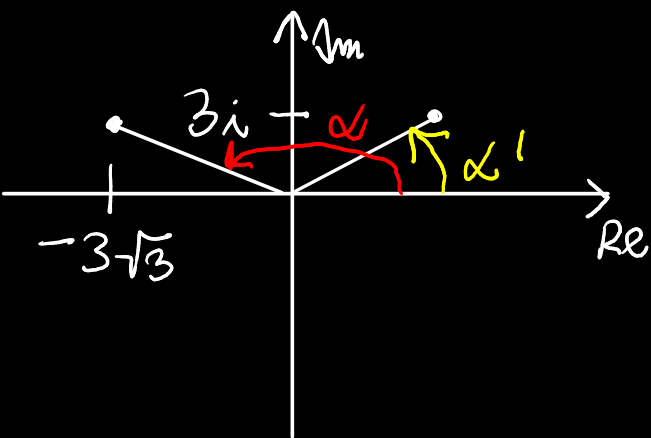
$$a = ((-3 - \sqrt{3}) + 3i)^{17}$$

$$|a| = \sqrt{a^2 + b^2}$$

$$|a| = \sqrt{9 \cdot 3 + 9} = 6$$

$$\cos \alpha' = \frac{|a_1|}{|a|} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} = \underline{\underline{30^\circ}}$$

$$\alpha = 180^\circ - 30^\circ = \underline{\underline{150^\circ}}$$



$$a = 6^{17} (\cos 150 \cdot 17 + i \sin 150 \cdot 17)$$

$$a = 6^{17} (\cos 2550^\circ + i \sin 2550^\circ)$$

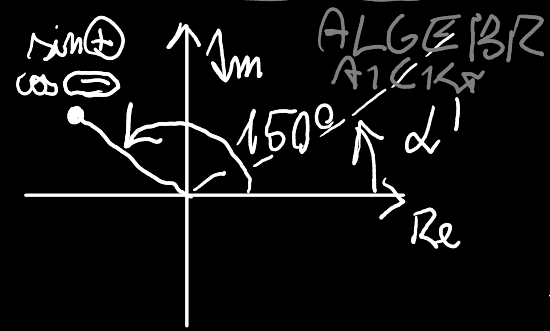
$$a = 6^{17} (\cos 30^\circ + i \sin 30^\circ)$$

$$a^{17} = 6^{17} \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right)$$

$$\begin{array}{r} 150 \\ \cdot 17 \\ \hline 1050 \\ 150 \\ \hline 2550 \end{array}$$

$$360 \cdot 7 = 2520$$

$$a = 5 e^{i \frac{5}{6} \pi} \quad a = 5 \cdot (\cos \frac{5}{6} \pi + i \sin \frac{5}{6} \pi)$$



$$a = 5 \cdot (\cos 150^\circ + i \sin 150^\circ) \leftarrow \text{GONIOMETRIK TRICKS}$$

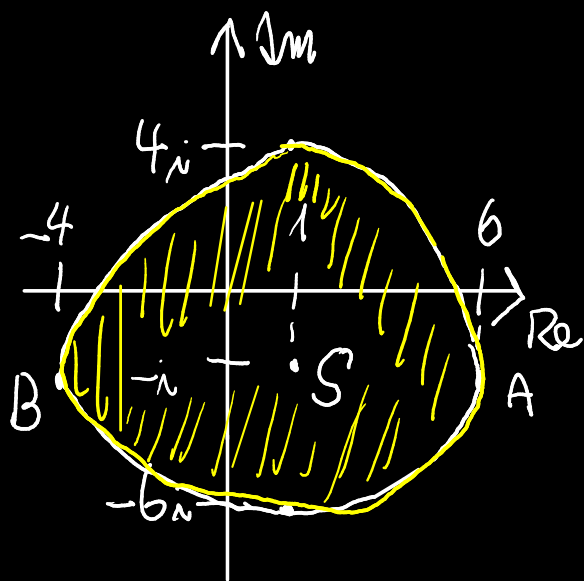
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$a = 5 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2} i \right) \leftarrow \text{ALGEBRAIC TRICKS}$$

$$a) |x - 1 + i| \leq |3 + 4i| \quad \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$$

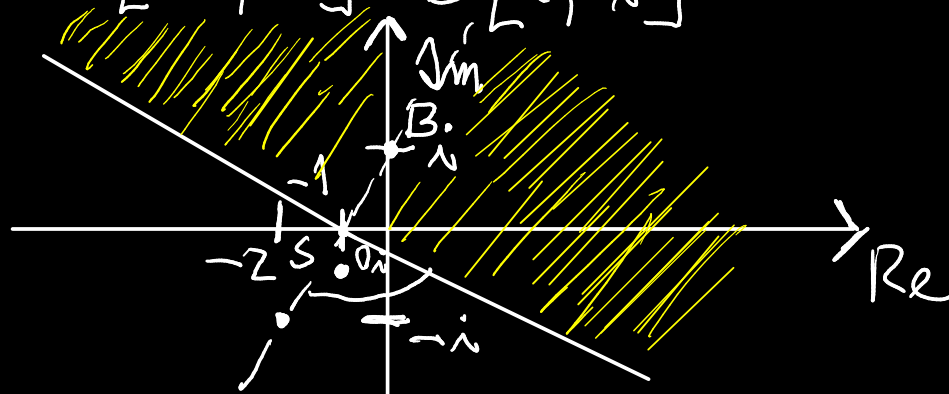
$$S [1; -i] \leq 5$$



- A [6; -1]
- B [-4; -1]
- C [1; -6i]
- D [1; 4i]

$$|x + 2 + i| > |x - i|$$

$$A [-2; -i] \quad B [0; i]$$



$$S_{AB} = [1; 0i]$$

$$2 \sin^2 x + 3 \cos x = 0$$

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \sin^2 x &= 1 - \cos^2 x \end{aligned}$$

$$2(1 - \cos^2 x) + 3 \cos x = 0$$

$$-2 \cos^2 x + 3 \cos x + 2 = 0$$

$$2 \cos^2 x - 3 \cos x - 2 = 0 \quad \Delta = \cos x$$

$$2 \Delta^2 - 3 \Delta - 2 = 0$$

$$D = b^2 - 4ac$$

$$D = 9 - 4 \cdot 2 \cdot (-2)$$

$$D = 9 + 16 = 25$$

$$\begin{aligned} x_1, x_2 &= \begin{cases} \nearrow x_1 = \frac{3+5}{4} = 2 \\ \searrow x_2 = \frac{3-5}{4} = -\frac{2}{4} = -\frac{1}{2} \end{cases} \end{aligned}$$

$$\Delta_1 = \cos(x) = 2 \quad * \quad \boxed{\cos(x) < 1}$$

$$\Delta_2 = \cos(x) = -\frac{1}{2} = 120^\circ$$

$$\underline{\underline{x = -\frac{1}{2} \cdot k; k \in \mathbb{Z}}}$$

$$ix^2 - 3x + 4i = 0$$

$$D = b^2 - 4ac$$

$$D = 9 - 4 \cdot i \cdot 4i$$

$$D = 9 - 16i^2$$

$$D = 9 + 16 \quad D = 25$$

$$x_1 = \frac{3+5}{2i} \cdot \frac{i}{i} = \frac{8i}{-2} = \underline{\underline{-4i}}$$

$$x_2 = \frac{3-5}{2i} \cdot \frac{i}{i} = \frac{3i-5i}{-2} = \frac{2i}{2} = \underline{\underline{i}}$$

Lk: $\odot x_1$

$$i(i^2) - 3i + 4i = 0$$

$$i(4i)^2 - 3(-4i) + 4i = 0$$

$$-i - 3i = -4i$$

$$\underline{\underline{-4i = -4i \checkmark}}$$

$$i \cdot (16i^2) + 12i + 4i = 0$$

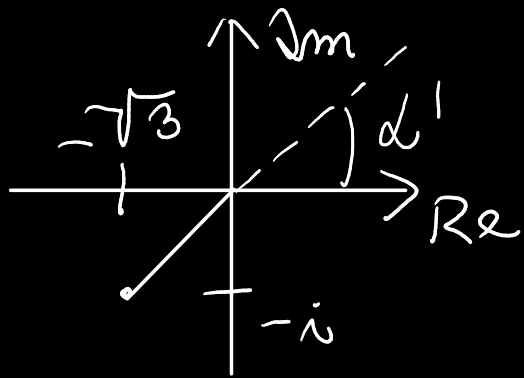
$$-16i + 12i + 4i = 0$$

$$\underline{\underline{0 = 0 \checkmark}}$$

$$a = -\sqrt{3} - i$$

$$|a| = \sqrt{3 + 1} = 2$$

$$\cos \alpha' = \frac{|a_{\text{re}}|}{|a|} = \frac{\sqrt{3}}{2} = 30^\circ = \frac{\pi}{6}$$



$$\underline{\underline{a = 2 \cdot (\cos 210^\circ + i \sin 210^\circ)}}$$

$$\underline{\underline{a = 2 e^{\frac{7\pi}{6}i}}}$$

$$3i^{17} + i^{90}$$

$$3i^{16 \cdot 1} + i^{88 \cdot 2}$$

$$3i - 1 \quad 6 + 8i$$

$$\underline{\underline{2 + 4i^{20} - 8i^{21}}}$$

$$2 + 4 - 8i$$

$$\underline{\underline{6 - 8i}} \cdot \frac{6 + 8i}{6 + 8i}$$

$$\underline{\underline{= \frac{18i + 24i^2 - 6 - 8i}{36 + 64} = \frac{10i - 30}{100} = -\frac{3}{10} + \frac{1}{10}i}}$$